Lecture 19: Center of Gravity

Center of Gravity (center of mass)

We have already used the idea of center of mass in the sense that when we write down Newton's Law of gravity, we treat the masses as though they were concentrated at a point, the center of mass of the planet, or sun. The concept of center of mass is more general that just spherical masses however. Consider taking two masses at the ends of a rod. If the masses are equal, this system is balanced when supported at its center, which is its center of gravity. The center of gravity for two masses is then found from,

$$m_1x_1 + m_2x_2 = m_1x_{cg} + m_2x_{cg} \tag{1}$$

Note that it does not matter where we choose the origin, as both sides are shifted in the same way when we change the origin. This equation generalizes to,

$$\sum_{i=1}^{n} m_ix_i = Mx_{cg} \tag{2}$$

where $M = \sum_i m_i$. The left hand side is the torque about an axis located a the origin of the co-ordinate system, and it can be replaced by the right hand side which is a single mass located at the center of gravity. This is really useful because we can treat complicated objects by a single weight located at their center of gravity. The center of mass along the y-axis and z-axis are calculated in a similar way.

$$\sum_{i=1}^{n} m_iy_i = My_{cg}; \quad \sum_{i=1}^{n} m_iz_i = Mz_{cg} \tag{3}$$

Important special case

For beams, plates, spheres and other "bodies" it is often simple to find the center of gravity by using symmetry arguments. For example the center of mass of a uniform beam is simply at its center. You need to know that to solve many of the CAPA problems. If you are given another object, like a person or a bear, we can replace the object by a single force located at its center of gravity. In this way we can locate the center the center of gravity of any object.

We will go through: Example 8.5, Example 8.6, Example 8.8, and a couple of the assigned CAPA problems.